FUNCTION

Before going through calculus, we have to under stand the concept of function. What is a function?

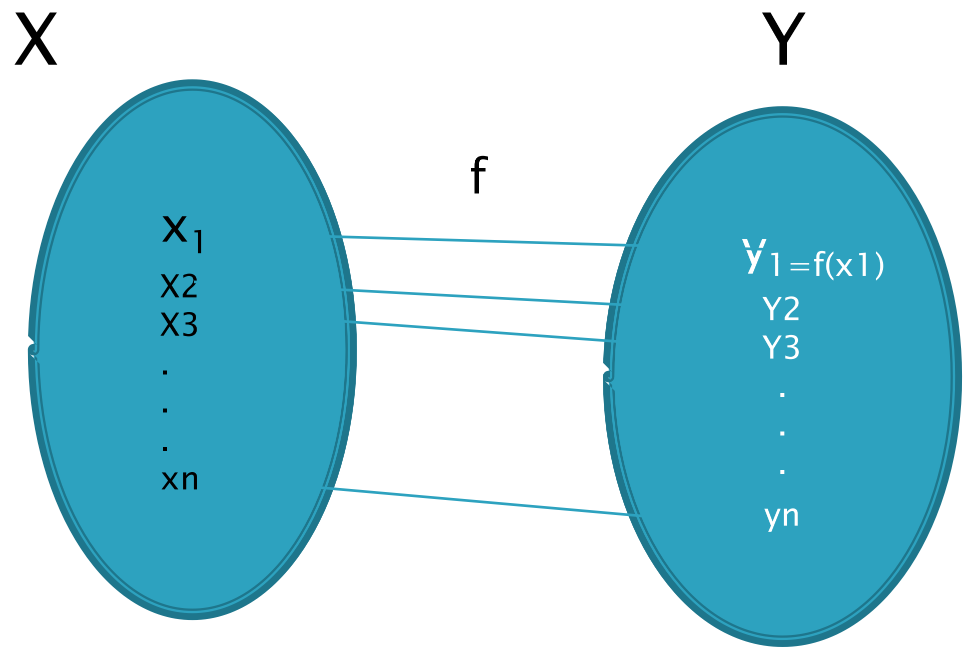
Definition: A relation f from X to Y is said to be function if it satisfies following two conditions,

i)Dom f=X ii)(x,y) Є f and (x,z)Єf implies that y=z

It to be remembered that f is a special type of relation.

$f \sqsubseteq X \times Y$ which satisfies above two conditions.

PICTORIAL REPRESENTATION



Dom f=X

Range $f \sqsubseteq Y$

$f \sqsubseteq XxY$

$y = f(x)$ Here y is the value of the function at x

x is the called independent variable and y is the dependent variable

TYPES OF FUNCTIONS

- Types of Functions:
- Polynomial function
- Greatest integer function
- Step function or staircase function
- Modulus function
- Remainder function
- Algebraic function
- Transcendental function

i)Trigonometric ii)Inverse Trigonometric
iii)Exponential iv)Logarithmic

- Constant function

EXPLANATION

- Polynomial function:
- General Mathematical Expression,

$p(x)=a_0+a_1x+a_2x^2+a_3x^3+\ldots\ldots\ldots\ldots+a_nx^n$

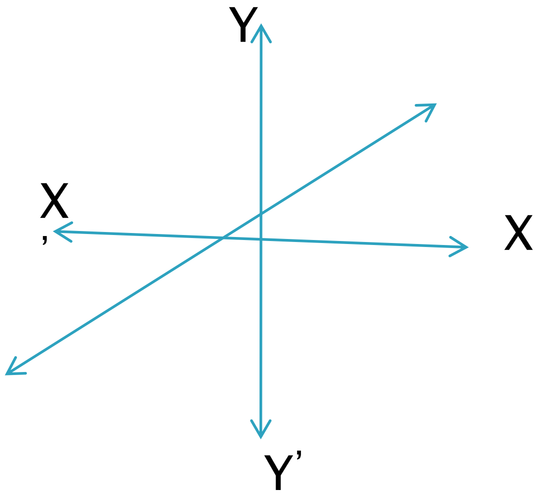
Where $a_{0,}a_{1,}a_{2,}a_{3,}\ldots\ldots\ldots\ldots\ldots\ldots a_n \in$ R and x is a variable and it also belongs to R.

The polynomial function may be of 1st degree, second degree, third degree and so on.

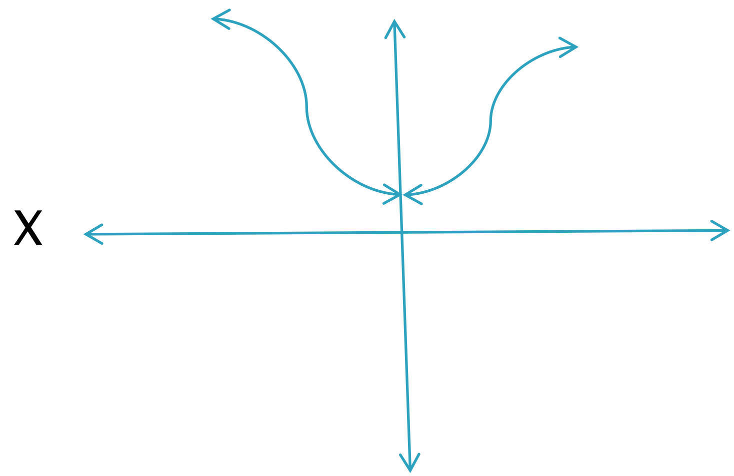
If the highest power variable x is 01,it is a linear function and all other polynomial function having degree 2,3,4 etc are non linear polynomial functions.

EXPLANATION

- General graphical representation of a linear and non linear polynomial function



$P(x)=a_0+a_1x$



$P_1(x)=a_0+a_1x+a_2x^2$

EXPLANATION

- Greatest Integer Function(Specific example)

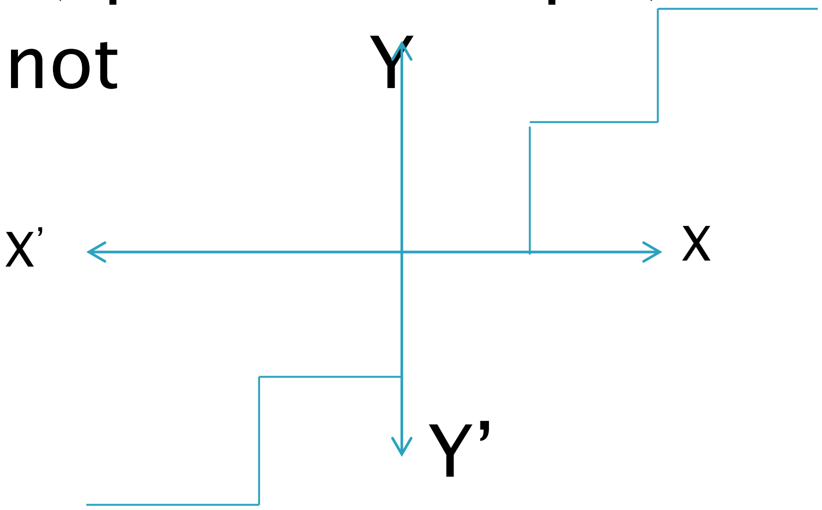
[x] is the greatest integer not exceeding x

$[x] = -1, \; -1 \leq x < 0$

$\quad = 0, \quad 0 \leq x < 1$

$\quad = 1, \quad 1 \leq x < 2$

$\quad = n, \quad n \leq x \leq n+1$

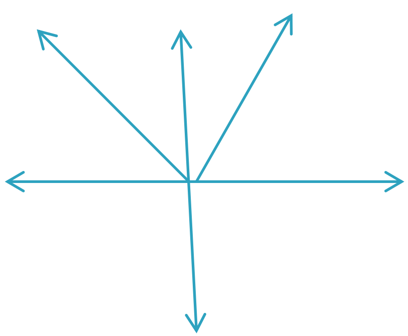


EXPLANATION

- Step function:
- The rate chart with post master for definite postage for a definite weight, p=f(w) is an example of step function which is known as post office function.
- Modulus function: It is defined as

$$y = |x| = x, x \geq 0$$
$$= -x, x < 0$$

1. The graph of the function:



EXPLANATION

- Remainder function:

$r_m(n)=r$ where $n \in Z$, m is any positive integer and the value of the function r is the remainder when n is divided by m. e.g $r_6(200)=2$

- Algebraic function : The ratio of two polynomial function is known as algebraic function or rational function. e.g

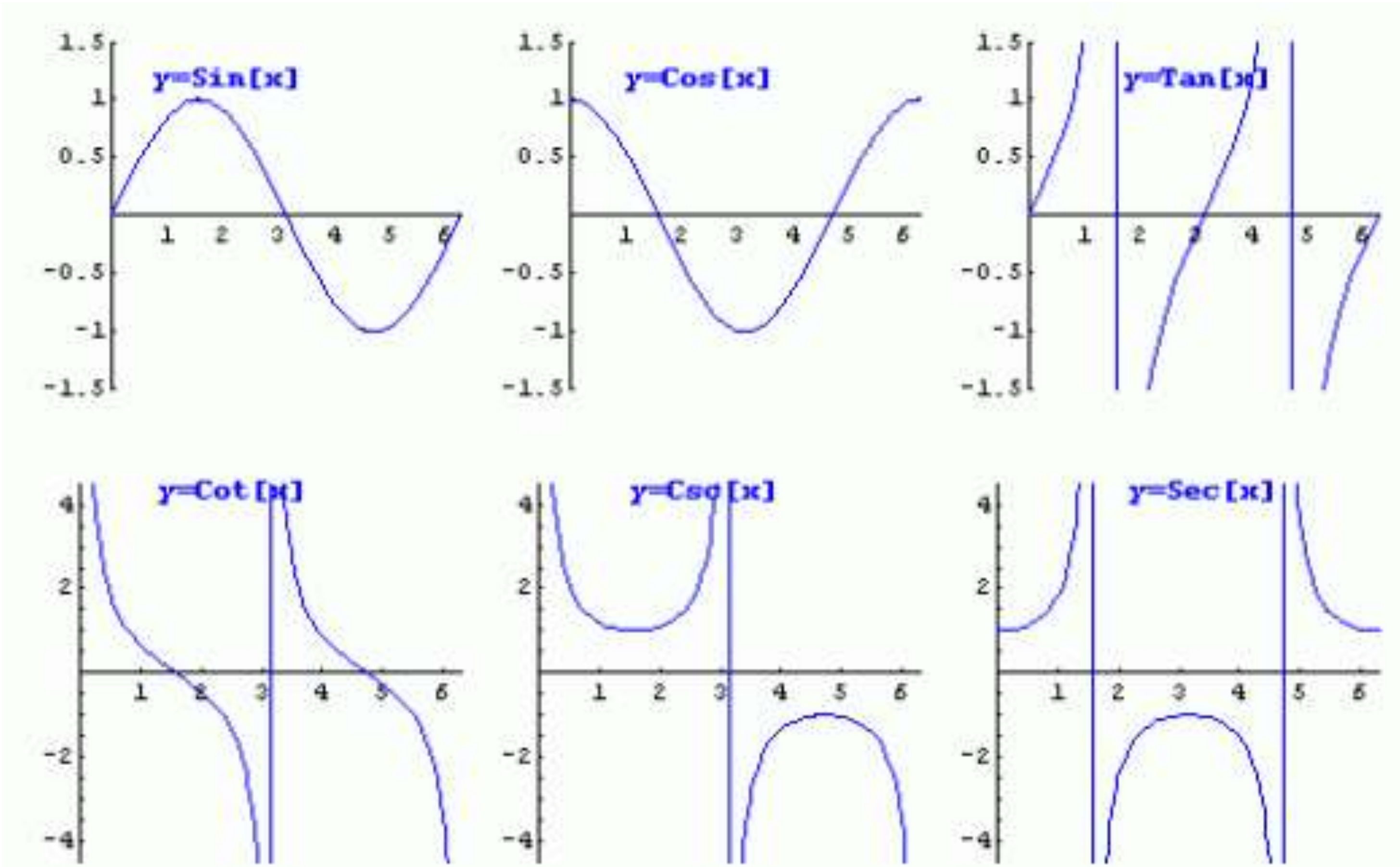
  $f(x)=x^2+x+1/x^3+2x^2+x+5$

- Transcendental function: It may be a Trigonometric, Inverse Trigonometric,Exponential or Logarithmic function

EXPLANATION

- Trigonometric Function:
  - ✓ Sin: $R \longrightarrow [-1,1]$ where R=Real number set
  - ✓ Cos: $R \longrightarrow [-1,1]$
  - ✓ Tan: $R' \longrightarrow R$, where $R' = R - \{(2n+1)\pi/2 : n \in Z\}$
  - ✓ Cotangent: $R'' \longrightarrow R$ where $R'' = R - \{n\pi : n \in Z\}$
  - ✓ Secant: $R' \longrightarrow R$
  - ✓ Cosecant: $R'' \longrightarrow R$
  - ✓ The graphical representations of the above functions are as follows,

EXPLANATION



Graphical representation of Trigonometric functions

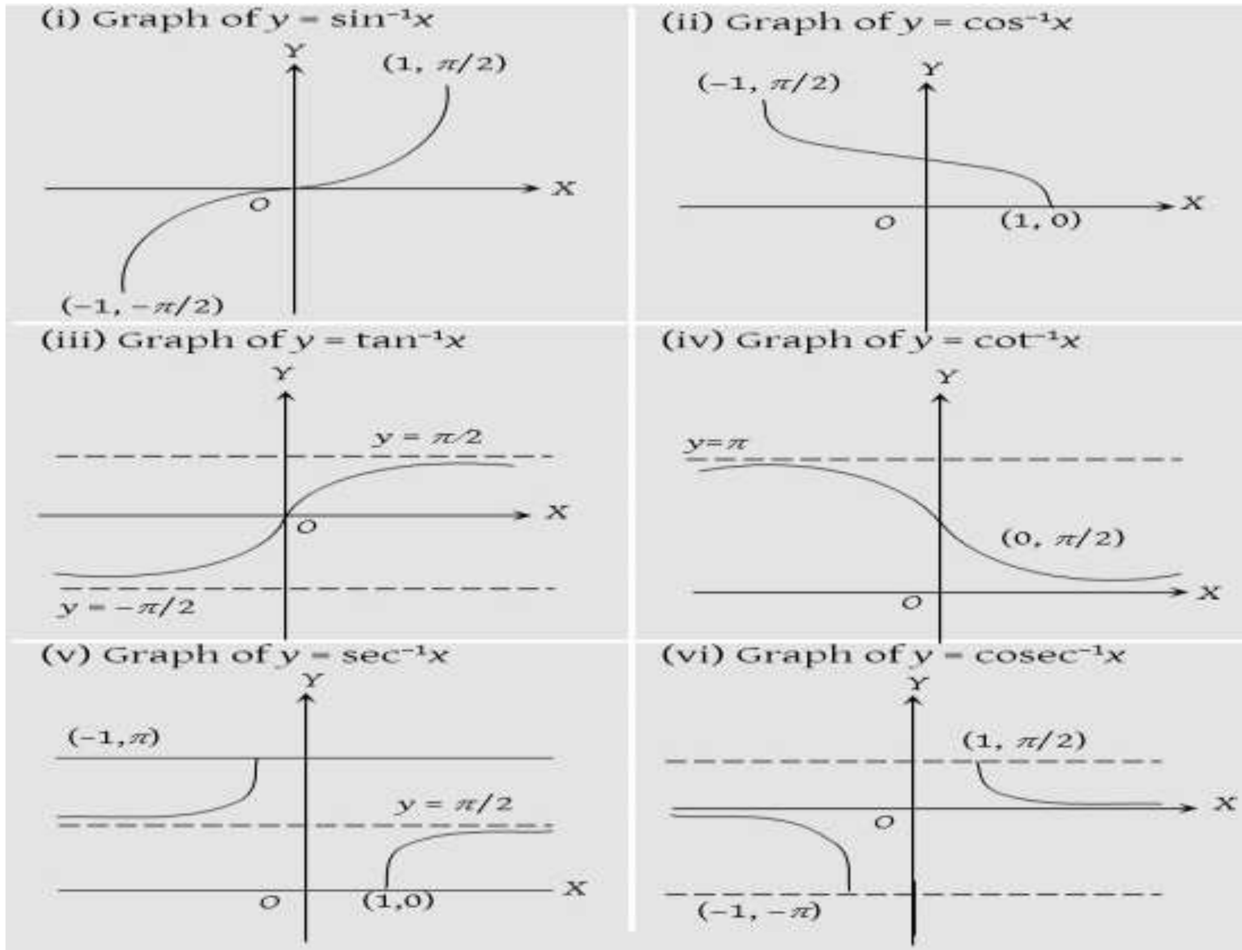
EXPLANATION

- Inverse Trigonometric functions:
- Sin^{-1}:[−1,1] ⟶[−л/2,л/2]
- Cos^{-1}:[−1,1]⟶[0,л]
- Tan^{-1}:(−∞,∞)⟶ (−л/2,л/2)
- Cot^{-1}:(−∞,∞) ⟶(0,л)
- Sec^{-1} (−∞,−1] ∪ [1,∞)⟶ (0,л/2]∪ [л/2,л)
- $Cosec^{-1}$: (−∞,−1] ∪ [1,∞)⟶ (−л/2,0]∪[0,л/2)

The graphical representation of the above inverse trigonometric function is as follows:

EXPLANATION

The graphical representation of Inverse trigonometric functions

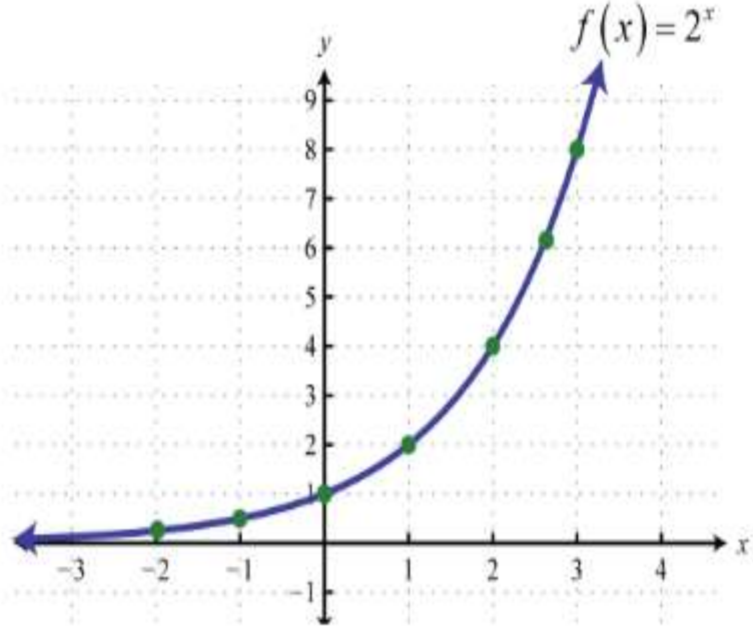


EXPLANATION

- Exponential Function:

$y=a^{x}, a>0, a\#1,$

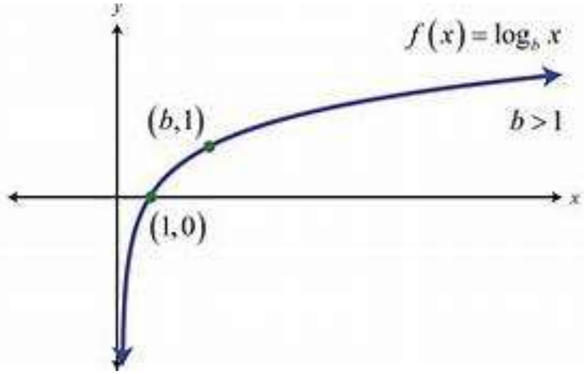
Here a =2



Graph of exponential function

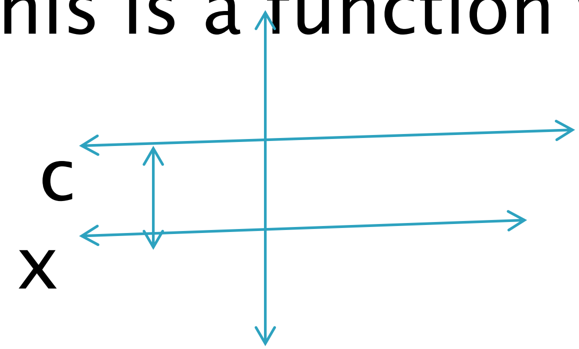
EXPLANATION

- Logarithmic Function:
- Y= $\log_b$ x where b>0,b#1



EXPLANATION

- Constant function:
- Y=f(x)=c, This is a function whose range is a singleton



- Odd and even function : A function said to be odd if f(-x)=-f(x) e.g $f(x)=x^3$, f(x)=sin x
- A function is said to be even if f(-x)=f(x),

e.g $f(x)=x^2$, f(x)=cos x

FUNCTION

We have told you that a function is a special type relation. It may be surjective(onto) or injective(one to one) or bijective(one to one correspondence).

To explain this let us define a function $f:X\rightarrow Y$, where X and Y are two non-empty sets.

EXPLANATION

- Range $f=R_f=f(X)=Y$, it is said to be surjective (onto)

Example: $f:R\rightarrow R$ such that $f(x)=2x$

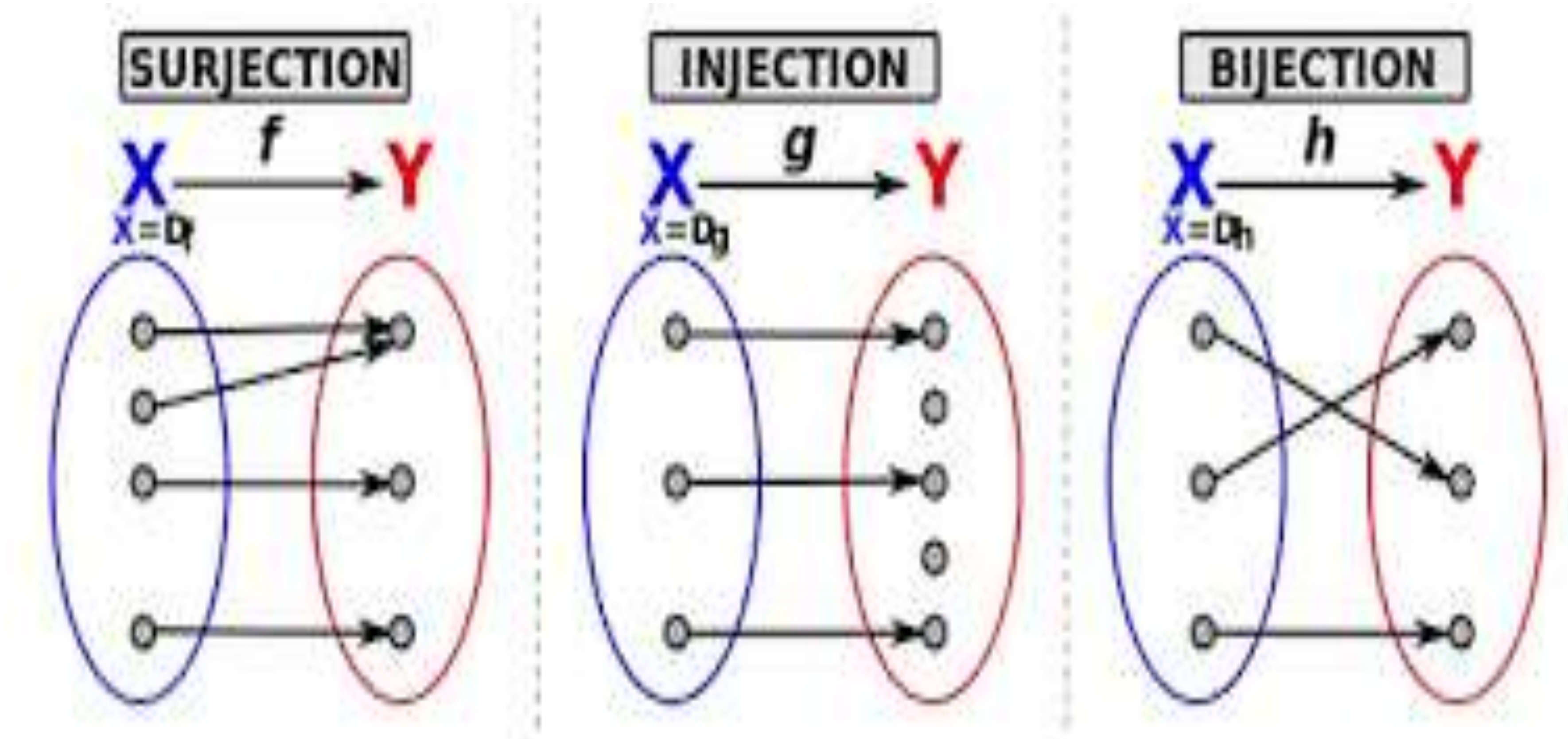
- $(x_1,y_1)\in f, (x_2,y_2)\in f \wedge x_1\neq x_2 \Rightarrow y_1\neq y_2$, it is said to be injective(one to one)

Example: $f:R\rightarrow R$ such that $f(x)=2x+1$

- f is both surjective and injective, then it is said to be bijective(one to one correspondence).

Examples: $f:R\rightarrow R$ such that $f(x)=3x-5$

PICTORIAL REPRESENTATION



INVERSE FUNCTION

- If f and g are two functions $f:R\rightarrow R$ such that $f(x)=y \Rightarrow x=g(y)$ and for every y there exists a x, then g is known as inverse function of f and denoted by f^{-1}. In terms of composition, we write, $g(f(x))=x$. If there exists an inverse function of a function, we say the function is invertible. The condition for invertibility of function is that it must be surjective and injective i.e it must be bijective.

Let $y=f(x)=\sin x$, $x \in R$ is a surjective function whose inverse $x= \sin^{-1}y$ is a relation but a function.

OPERATION ON FUNCTIONS

Two functions are defined between two given sets, then their addition, multiplication and quotients being a function may be defined accordingly.

Let $f,g:X \rightarrow R$, then addition of two functions $(f+g)$ is also a function and can be defined as

$$(f+g)(x)=f(x)+g(x), x \in X$$

Multiplication of two functions(fg) can be defined as $(fg)(x)=f(x)g(x), x \in X$

Quotient of two functions $f/g=f(x)/g(x), x \in X$ provided that $g(x) \neq 0$

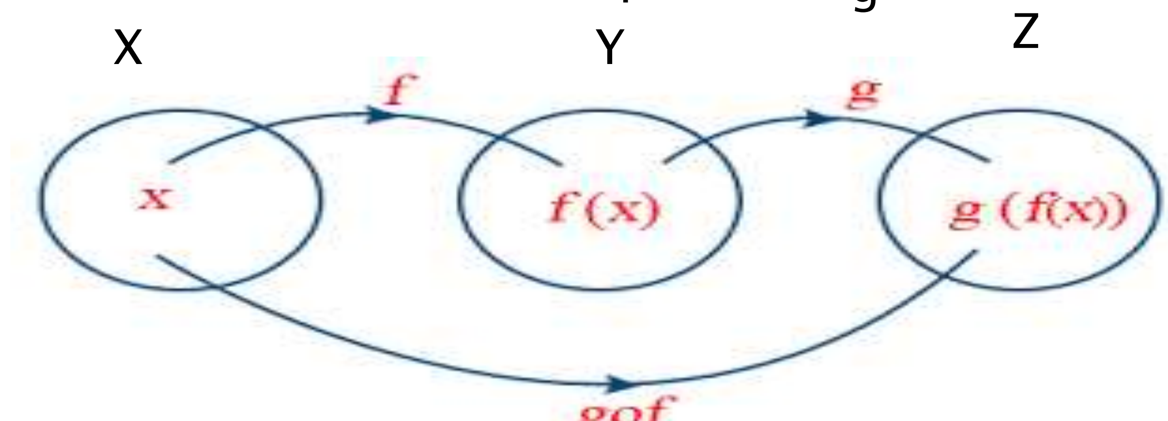
COMPOSITION FUNCTION

We may get a new function with the help of two given functions, which is known as composition function.

Let $f:X \rightarrow Y$ and $g:Y \rightarrow Z$, the composition of functions f and g written as gof is defined as

$gof(x) = g(f(x)), x \in X$ provided $R_f \sqsubseteq Y = D_g$

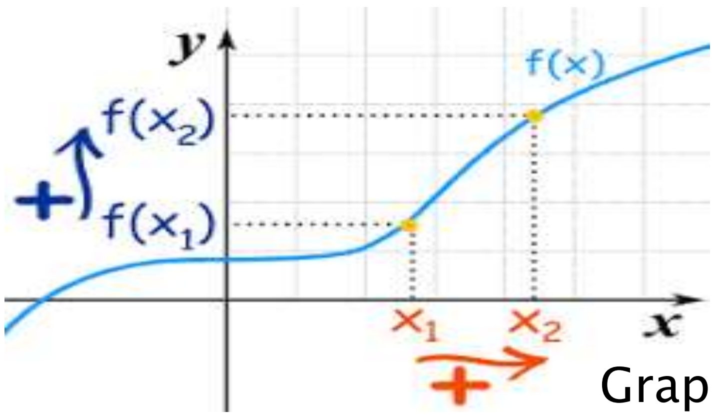
Composition of function is associative but not commutative

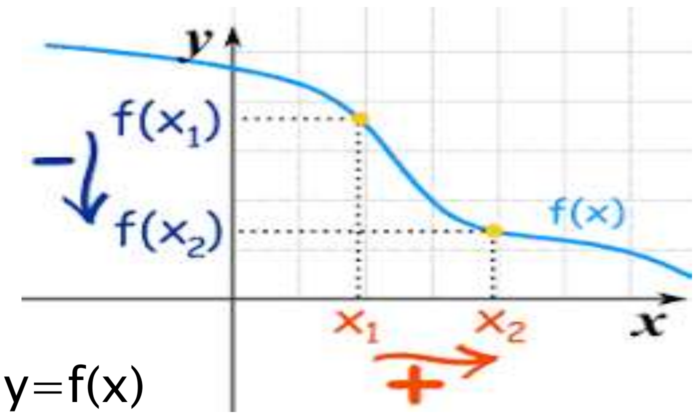


(Diagram of composition function)

CONCLUSION

Every function y=f(x) on R may be graphically represented by its characteristic graph.





Graph of y=f(x)

A function f on R is monotonically increasing, $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$ and it is monotonically decreasing, $x_2 > x_1 \Rightarrow f(x_2) < f(x_1$